Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3201784670

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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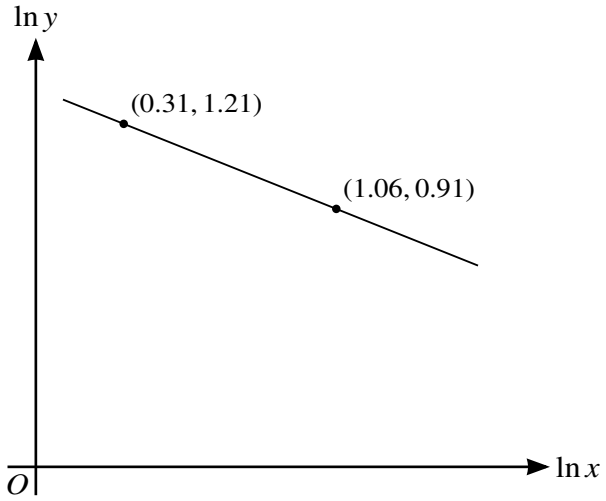
1 Solve the inequality $|2x + 3| > 3|x + 2|$. [4]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2 - 3i| \leqslant 2$ and $\arg z \leqslant \frac{3}{4}\pi$. [4]

3



The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (0.31, 1.21) and (1.06, 0.91), as shown in the diagram.

Find the value of n and find the value of C correct to 2 decimal places. [5]

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4 The parametric equations of a curve are

$$x = 1 - \cos\theta, \qquad y = \cos\theta - \tfrac{1}{4}\cos 2\theta.$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2\sin^2\left(\tfrac{1}{2}\theta\right)$. [5]

5 The angles α and β lie between $0°$ and $180°$ and are such that

$$\tan(\alpha + \beta) = 2 \quad \text{and} \quad \tan\alpha = 3\tan\beta.$$

Find the possible values of α and β. [6]

6 Find the complex numbers w which satisfy the equation $w^2 + 2iw^* = 1$ and are such that $\text{Re}\, w \leqslant 0$. Give your answers in the form $x + iy$, where x and y are real. [6]

7 **(a)** By sketching a suitable pair of graphs, show that the equation $4 - x^2 = \sec \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 2. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{4 - \sec \frac{1}{2}x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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8 **(a)** Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3 + 4x^2 + 2x + 7}{4x^2 + 1}\,\mathrm{d}x$. [5]

9 The variables x and y satisfy the differential equation

$$(x + 1)(3x + 1)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = 3$, giving your answer in a simplified form. [9]

10 The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ respectively. The line l has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$.

(a) Find a vector equation for the line through A and B. [3]

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(b) Find the acute angle between the directions of AB and l, giving your answer in degrees. [3]

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(c) Show that the line through A and B does not intersect the line l. [4]

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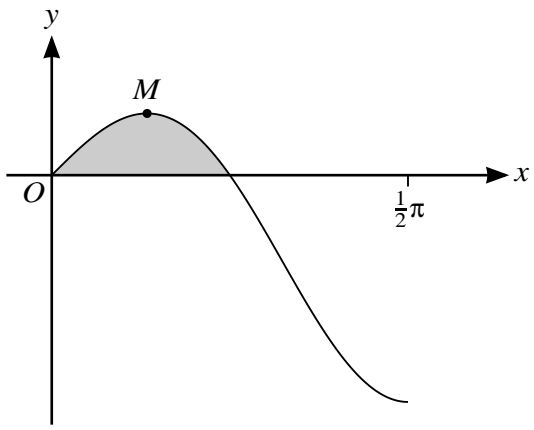

The diagram shows the curve $y = \sin x \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 significant figures. [6]

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(b) Using the substitution $u = \cos x$, find the area of the shaded region enclosed by the curve and the x-axis in the first quadrant, giving your answer in a simplified exact form. [5]

Additional Page

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